CAMBRIDGE INTERNATIONAL EXAMINATIONS

Pre-U Certificate

MARK SCHEME for the May/June 2013 series

9795 FURTHER MATHEMATICS

9795/02 Paper 2 (Further Application of Mathematics), maximum raw mark 120

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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1	**(i)**	Use of N(25,25) (in either part) $z = \dfrac{15.5 - 25}{5} = -1.9$ P (≤ 15) = 0.0287	**M1** **M1A1** **A1** **[4]**
	(ii)	$\Phi(z) = 0.95 \Rightarrow z = 1.645$ (Allow [1.64,1.65]) $x = 0.5 + 25 + 5 \times 1.645 = 33.725$ (Allow for ± 0.5) ⇒ 34 narrowboats required. (CAO)	**B1** **B1** **B1** **[3]**
2	**(i)**	$\overline{B} \sim \left(156, \dfrac{64}{9}\right)$, $\overline{G} \sim \left(160, \dfrac{49}{16}\right)$ (Can be implied by working.) $\overline{B} - \overline{G} \sim N\left(-4, \dfrac{1465}{144}\right)$ $z = \dfrac{0-(-4)}{\sqrt{\dfrac{1465}{144}}} = 1.254$ $1 - \phi(1.254) = 0.1049 = 0.105(3\text{sf})$ (AWRT 0.105)	**B1B1** **B1M1A1** **M1A1** **A1** **[8]**
	(ii)	Samples are taken from underlying normal distributions ⇒ distributions of sample means are normal.	**B1** **[1]**
3	**(i)**	$\text{P}(X > 6 \mid X > 3) = \dfrac{(1-0.7622)}{(1-0.2650)}$ $= 0.324$	**M1A1** **A1** **[3]**
	(ii)	$\text{P}(\leqslant 1) = \text{e}^{-\lambda}(1-\lambda) = \dfrac{1}{2}$ $\Rightarrow \text{e}^{\lambda} = 2(1+\lambda) \Rightarrow \lambda = 1\text{n}\{2(1+\lambda)\}$ (AG)	**M1A1** **A1** **[3]**
	(iii)	$\lambda_0 = 1.6$ or 1.7 $\lambda_{r+1} = \ln\{2(1+\lambda_r)\}$ $\lambda = 1.678\,(3\text{dp})$ (Award B2 for correct answer with no working.)	**B1** **M1** **A1** **[3]**

4	(i)	P: $\bar{x}_1 = 4.0125$ S.D.$_P$ = 0.73218 … or 0.78273 … (or variance) Q: $\bar{x}_2 = 2.55$ S.D.$_Q$ = 0.39895 … or 0.43703 … (or variance)	B1 B1
		$\hat{\sigma}^2 = \dfrac{4.28875 + 0.955}{8+6-2} = 0.43697$ $t_{12}(0.95) = 1.782$	M1A1 B1
		90% confidence limits are: $1.4625 \pm (1.782 \times 0.66104... \times \sqrt{8^{-1} + 6^{-1}})$ (ft on t value.) 90% confidence interval is (0.826 , 2.10) (Accept 0.827 from t =1.78.)	M1A1√ A1 [8]
	(ii)	Distributions of broadband speeds are normal. The populations have a common variance.	B1 B1 [2]
5	(i) (a)	$k(5+3+2) = 1 \Rightarrow k = \dfrac{1}{10}$	B1 [1]
	(b)	Modal value is –1.	B1 [1]
	(ii) (a)	$G_y(t) = \dfrac{1}{100}(5t^{-1} + 3 + 2t^2)^2$ (ft on k value and also in (b):1st two lines.)	B1√ [1]
	(b)	$G_y'(t) = \dfrac{1}{50}(5t^{-1} + 3 + 2t^2)(-5t^{-2} + 4t)$	M1A1√
		$E(Y) = G_y''(1) = -\dfrac{10}{50} = -\dfrac{1}{5}$	B1
		$G_y''(t) = \dfrac{1}{50}\{(-5t^{-2} + 4t)^2 + (5t^{-1} + 3 + 2t^2)(10t^{-3} + 4)\}$ $G_y''(1) = \dfrac{1}{50}(1 + 10 \times 14) = \dfrac{141}{50} = \sigma^2 + \dfrac{1}{25} + \dfrac{1}{5}$ (B1 for their $\sigma^2 + \mu^2 = \mu$.)	M1A1√ A1B1
		$\Rightarrow \sigma^2 = \dfrac{129}{50} = 2.58$ (AWRT 2.58)	A1 [8]
6	(i)	$F(x)\begin{cases} 0 & x < 0 \\ \dfrac{x}{2} & 0 \le x \le 2 \\ 1 & x > 2 \end{cases}$	B1B1 [2]
	(ii)	$Y = X(8-X) = 8X - X^2 \Rightarrow 16 - Y = 16 - 8X + X^2 = (4-X)^2$ (AG) $G(y) = \dfrac{1}{2}\{4 - \sqrt{16-y}\}, 0 \leqslant y \leqslant 12$ $\therefore g(y) = G'(y) = \dfrac{1}{4\sqrt{16-y}}, 0 \leqslant y \leqslant 12$ (AG)	M1A1 B1B1 B1 [5]

	(iii)	G (Median) = $\frac{1}{2} \Rightarrow$ Median = 7	M1A1 [2]
	(iv)	$\mathrm{E}(Y) = \int_0^{12} \frac{y}{4\sqrt{16-y}}\,\mathrm{d}y$ Let $16 - y = u^2$ (or any other valid substitution) $\mathrm{E}(Y) = \int_2^4 (8 - \frac{1}{2}u^2)\,\mathrm{d}u$ $\mathrm{E}(Y) = \left[8u - \frac{u^3}{6}\right]_2^4 = \left[32 - \frac{32}{3}\right] - \left[16 - \frac{4}{3}\right] = \frac{20}{3}$ Or: $\left[-\frac{1}{2}y(16-y)^{\frac{1}{2}}\right]_0^{12} + \int_0^{12}(16-y)^{\frac{1}{2}}\,\mathrm{d}y$ $= [-12] + \left[-\frac{1}{3}(16-y)^{\frac{3}{2}}\right]_0^{12} = \frac{20}{3}$ Or: $\mathrm{E}(Y) = \int_0^2 \frac{1}{2}(8x - x^2)\,\mathrm{d}x$ $= \left[2x^2 - \frac{1}{6}x^3\right]_0^2 = \frac{20}{3}$	B1 M1A1 M1A1 [5] (M1A1) (M1A1) (M1A1) (M1A1)
7		Work against gravity: $3 \times 10^3 \times 25 \times 10 = 750\,000$J Let speed of delivery be v ms^{-1}. $\pi \times 0.05^2 \times 60v = 3$ $\Rightarrow v = \frac{3}{\pi \times 0.05^2 \times 60} = 6.366$ or $\frac{20}{\pi}$ Kinetic energy $\frac{1}{2} \times 3 \times 10^3 \times 6.366^2 = (60792)$ Power $= \frac{750000 + 60792}{60 \times 1000} = 13.5$kW	M1A1 M1A1 A1 B1 M1A1 [8]
8	(i)	$80 = \omega^2(a^2 - 4)$ $64 = \omega^2(a^2 - 5)$ Dividing: $\frac{5}{4} = \frac{a^2 - 4}{a^2 - 5} \Rightarrow a^2 = 9 \Rightarrow a = 3$ $\omega = 4$, $T = \frac{2\pi}{4} = \frac{\pi}{2}$ seconds.	B1 B1 M1A1 A1A1 [6]
	(ii)	$t_1 - t_2 = \frac{1}{4}\sin^{-1}\left(\frac{2}{3}\right) - \frac{1}{4}\sin^{-1}\left(\frac{-\sqrt{5}}{3}\right) \left(= 0.1824... + 0.2102...\right)$ $\Rightarrow t_1 - t_2 = 0.393$, or $\frac{1}{8}\pi$, seconds (3sf)	M1A1 A1 [3]

9	(i)	$v = 4,\ v\dfrac{dv}{dx} = -2 \Rightarrow k = \dfrac{1}{16}$	B1
		$-kv^3 = 2v\dfrac{dv}{dx}$	M1
		$\int \dfrac{1}{16}dx = 2\int -v^{-2}dv$	M1
		$\dfrac{x}{16} = \dfrac{2}{v} + c$	A1
		$x = 0,\ v = 4 \Rightarrow c = -\dfrac{1}{2}$	A1
		$\dfrac{2}{v} = \dfrac{x}{16} + \dfrac{1}{2} \Rightarrow \dfrac{1}{v} = \dfrac{x+8}{32}$ (AG)	A1 [6]
	(ii)	$v = \dfrac{dx}{dt} = \dfrac{32}{x+8}$	M1
		$\int_0^t 32dt = \int_0^8 (x+8)dx$	M1A1
		$32t = \left[\dfrac{x^2}{2} + 8x\right]_0^8 = 96$	
		$\Rightarrow t = 3$	A1 [4]
		Or	
		$-\dfrac{v^3}{16} = 2\dfrac{dv}{dt} \Rightarrow \int \dfrac{1}{16}dt = \int 2v^{-3}dv$	
		$\Rightarrow -\dfrac{t}{16} + c = -v^{-2}$	(M1)
		$t = 0,\ v = 4 \Rightarrow c = -\dfrac{1}{16} \Rightarrow -\dfrac{t}{16} - \dfrac{1}{16} = -\dfrac{1}{v^2}$	(M1)
		$\Rightarrow t + 1 = \dfrac{16}{v^2} \Rightarrow v^2 = \left(\dfrac{16}{t+1}\right)$	(A1)
		$x = 8,\ v + 2 \Rightarrow 4 + \dfrac{16}{t+1} \Rightarrow t = 3$	(A1)

10	**(i)**	Uses A's velocity perpendicular to ${}_AV_B$ for closest approach. Calculates relevant angle from (12, 16, 20) ~ (3, 4, 5) triangle. e.g. $\cos^{-1}\left(\frac{4}{5}\right) = 36.87°$ Finds bearing: 300º + 36.87º = 337º (nearest degree).	**M1** **M1A1** **A1** **[4]**
	(ii)	Magnitude of ${}_AV_B = \sqrt{20^2 - 16^2} = 12$ Uses distance triangle to find length of travel along relative path: 15cos(120 – 53.13)° = 5.892 km $\text{Time} = \frac{5.892}{12} \times 60 = 29.5$ minutes	**M1A1** **M1A1** **M1A1** **[6]**
11	**(i)**	Conservation of energy: $50.10\left(\frac{4}{5} - \frac{1}{2}\right) = \frac{1}{2} \times 5v^2$ $v^2 = 60 \Rightarrow v = \sqrt{60} \quad (= 7.75)$ Speed is 7.75 ms^{-1}. Resolving along string: $T - 50 \times \frac{4}{5} = \frac{5}{10} \times 60$ $\Rightarrow T = 70$ Tension is 70N	 **M1A1** **A1** **A1** **M1A1** **A1** **[7]**
	(ii)	Acceleration towards centre: $\frac{60}{10} = 6$ Newton II along tangent: $50 \times \frac{3}{5} = 5a \Rightarrow a = \frac{30}{5} = 6$ Magnitude of acceleration $6\,sec\,45° = 6\sqrt{2}\text{ms}^{-2}$ (AG) (CWO)	 **B1** **M1A1** **A1** **[4]**

12	**(i)**	Let speed on *AP* be u and speed on *PB* be v.	
		Momentum: $u\cos\theta = v\sin\theta$ (Method mark can be awarded in part (ii), if not here)	**M1A1**
		NEL: $eu\sin\theta = v\cos\theta$ (Method mark can be awarded in part (ii), if not here)	**M1A1**
		$e\dfrac{\sin\theta}{\cos\theta} = \dfrac{\cos\theta}{\sin\theta} \Rightarrow e = \dfrac{1}{\tan^2\theta}$	**M1**
		$\Rightarrow e = \dfrac{1}{9}$	**A1** **[6]**
	(ii)	Momentum: $u\cos\theta = v\cos(135° - \theta) = v\sin(\theta - 45°)$	**B1**
		NEL: $\dfrac{2}{3}u\sin\theta = v\cos(\theta - 45°)$	**B1**
		Dividing: $\dfrac{2}{3}\tan\theta = \dfrac{\cos\theta\cos45° + \sin\theta\sin45°}{\sin\theta\cos45° - \cos\theta\sin45°} = \dfrac{1+\tan\theta}{\tan\theta - 1}$	**DM1**
		$\Rightarrow 2(t^2 - t) = 3 + 3t$ (where $t = \tan\theta$) $\Rightarrow 2t^2 - 5t - 3 = 0$	**A1**
		$(2t+1)(t-3) = 0 \Rightarrow t = 3$ or $-\dfrac{1}{2}$ Since θ is acute, $\theta = \tan^{-1}3$	**DM1A1** **[6]**